RESEARCH ARTICLE OPEN ACCESS

Modeling and Estimation of Stationary and Non-stationary Noises of Rubidium Atomic Clock

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Abstract—
Noise estimation of atomic clock is one of the important research areas in the field of atomic clock development and application. Most of the atomic clocks are having random-stochastic noises and periodic noises due to temperature variation. Random-stochastic noises have a well identified signature in time domain but periodic noises are difficult to analyze in time domain. However, in this paper, an effort is made to identify and analyze the deterministic trends of both random-stochastic noises and periodic noises due to variation in temperature using an alternate approach of least-squares normalized-error (LSNE) regression algorithm.

A MATLAB based application with graphical user interface (GUI) is developed to estimate and analyze random-stochastic noises and periodic noises and re-estimate the stability of rubidium atomic clock after removing these noises from the raw phase data. The estimation of stationary noises are done using Allan variance from time domain data and noise profile is calculated using curve fit method. The estimation of periodic noises due to temperature variation is carried in frequency domain through spurious analysis of the frequency data of atomic clock.

Index Terms— LSNE, Rb, GUI, FP, WF, RWF

I. INTRODUCTION

Today the Atomic clocks are backbone of navigation systems. The performances of these atomic clocks are limited by inherent noises present in the corresponding clocks. These limitations can be assessed with the help of modeling of noise process present in the clock. The most important parameter of an atomic clock is stability, which is mostly affected by different types of noises, inherently present in clock. There are various types of noises present in clock, but can be broadly categorize in stationary and non-stationary category. The major contributors of deterministic trend non-stationary noises are temperature variation, power supply variation and magnetic sensitivity of clock. In this paper, we have estimated a periodic noise due to temperature variation of atomic clock. Similarly random-stochastic noises are the major contributor of stationary noises. The Allan Variance technique [1] has long been applied to the characterization of different noises in atomic clock. In this paper, an attempt is made to present least-squares normalized-error (LSNE) regression method for interpreting different types of noise present in clock data. Although least squares normalized-error regression method is already used for estimation of different noises in various subsystems [2][3][4]&[5],but for atomic clock noise estimation it was never applied. In this paper we have presented a new approach using least squares normalized-error regression method for noise estimation of atomic clock.

II. ESTIMATION OF RANDOM–STOCHASTIC NOISES

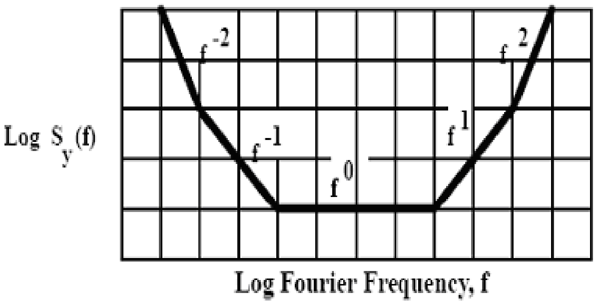
In the field of satellite navigation, various simulation tools are developed for the design and validation of atomic clock stability, however the clock errors due to noise is required to estimate in order to measure the clock stability accurately. Therefore to predict the clock behavior accurately, precise clock errors modeling are required. As discussed-earlier, random-stochastic noises come in
the stationary noises as well as non-stationary noise. There are five types of random stochastic noises present in atomic clock named as WF (White noise on Frequency), WP (White noise on Phase), FF (Flicker noise on Frequency), FP (Flicker noise on Phase) and RWF (Random Walk noise on the Frequency). In this paper, we have modeled rubidium atomic clock where WF, FF and RWF are considered to be the dominating noise sources throughout the frequency [6]. The clock model is generated using MATLAB and SIMULINK to predict the clock stability.
It is well documented that the instability of most frequency sources can be modeled by a combination of power-law noises [6] having a spectral density of their fractional frequency fluctuations of the form $S_y(f) = kf^{\alpha}$, where f is the Fourier or sideband frequency in hertz, and α is the power law exponent. Table 1 [7] shows typical value of the power law exponent for clock noises. Figure 1 shows the theoretical (or usual

behavior) characteristics of power spectral density of the clock noises.

TABLE I: POWER LAW EXPONENT FOR CLOCK NOISES

Noise Type	α (Power Law Exponent)
White PM(WP)	2
Flicker PM(FP)	1
White FM(WF)	0
Flicker FM(FF)	-1
Random Walk FM(RWF)	-2

Figure 1: Power Spectral Density of Clock noise error sources [6]



Time domain frequency stability measure (Allan Deviation) is related to the spectral density of the fractional frequency fluctuations by the relationship given below [8]:

$$\sigma^2(M,T,\tau) = \int_0^\infty S_Y(f).|H(f)|^2\, df \quad (1)$$

Where, σ^2(M,T,t) is the M - sample Allan Variance for time t and sampling period T. S_y(f) is the power spectral density of fractional frequency fluctuations. $|H(f)|^2$ is the transfer function of Allan (two sample) time domain stability, which is given in equation 2.

$$|H(f)|^2 = 2*[sin^4(\pi t f / (\pi t f)^2)] \quad (2)$$

Theoretical Allan deviation plot for the Atomic Clock noises is given in the figure 2.

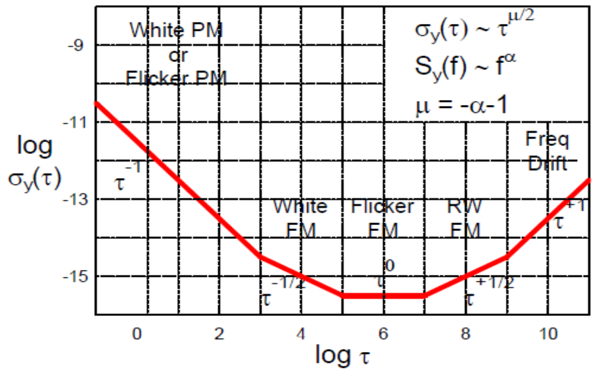


Figure 2: Allan Deviation (σ_y(t)) of the Clock Noise Error Sources [9]

A. Proposed algorithm

The regression algorithm proposed in this paper begins with the Allan Variance calculation from time domain data as indicated by Tehrani [10]. The algorithm assumes each noise has different signature. This algorithm estimates the signature of different random noises present in each decade of log-log plot. The slope of different noise in the Allan variance against time (tau) is mentioned Table 2 [6]:

These noises are uncorrelated. Hence the equation describing the total Allan Variance is [11].

$$\sigma_{Tot}^2 = \sigma_{WPM}^2 + \sigma_{FPM}^2 + \sigma_{WFM}^2 + \sigma_{FFM}^2 + \sigma_{RWFM}^2 \quad (3)$$

Where σ_{Tot}^2 is the total Allan Variance, σ_{WPM}^2 is the variance due to white PM noise, σ_{FPM}^2 is variance due to flicker PM NOISE, σ_{WFM}^2 is the variance due to white FM noise , σ_{FFM}^2 is due to flicker FM noise and σ_{RWFM}^2 is due to random walk FM noise. However in case of rubidium clock the total Allan Variance is only affected by white FM, flicker FM and random walk FM noise [6].

TABLE 3: RELATIONSHIP BETWEEN PSD AND ALLAN VARIANCE

Type of noise	Slope in Log domain
White PM	τ^{-2}
Flicker PM	τ^{-2}
White FM	τ^{-1}
Flicker FM	τ^{-0}
Random Walk FM	τ^{+1}

The proposed algorithm begins by normalization of data generated by clock measurement. Since this generated data span several Kbytes, it was desirable to normalize the errors to the localized magnitude of the data. Sargent and Wyman [11] accomplished this by calculating the logarithm of the data. In this paper we have used an alternate method of normalization by dividing the data by the value of the curve fit at the associated measurement time.
The error equation is given by

$$\varepsilon = \frac{\sigma_{Tot}^{2} - \sigma_{Tot}^{\wedge 2}}{\sigma_{Tot}^{\wedge 2}}$$

(4)

Where ε is defined as the "normalized error" and $\sigma_{Tot}^{\wedge 2}$ is the estimate of the total Allan Variance for a given measurement time. However, the estimate of the total Allan Variance from the curve fit is not available initially. Therefore, a modified definition of the normalized error is created using the actual data at that value of measurement time or

$$\varepsilon = \frac{\sigma_{Tot}^{2} - \sigma_{Tot}^{\wedge 2}}{\sigma_{Tot}^{2}} \qquad (5)$$

This modified definition of the normalized error allows for a simplified derivation of the least-squares normalized error (LSNE) regression.
The derivation of the least-squares normalized error curve fit follows the least-squares algorithm very closely. The error IS shown in equation (5) can be generalized as

$$\varepsilon = \frac{y - \ddot{y}}{y} \qquad (6)$$

Where the measured variance in (5) has been replaced by *y* and the value of the curve fit has been replaced by ÿ. The equation for ÿ can be generalized as [12]

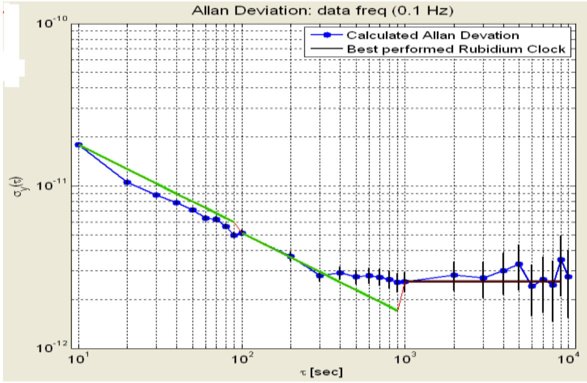
$$\ddot{y} = C_m \tau^m + C_n \tau^n + C_p \tau^p + C_g g \qquad (7)$$

It is desired to minimize the sum of the squares of the normalized error from the curve fit by solving for the coefficients that minimize this error. The sum of the squares of the normalized error is defined as

$$\sum_{\tau_{\min}}^{\tau_{\max}} \varepsilon^2 = \sum_{\tau_{\min}}^{\tau_{\max}} \left[\frac{y - \ddot{y}}{y} \right]^2 \qquad (8)$$

The proposed algorithm divides the complete measurement time in decade of log-log plot. For each decade sum of the normalized error is minimized .The predicated slope of each decade is map to one of noise characteristic, whose behavior close to predicated slope. Figure3 shows the estimated curve fit and estimated noise present using proposed method.

Figure 3: Estimated curve fit of different noises in Allan variance plot



III. ESTIMATION OF PERIODIC ATOMIC CLOCK NOISES

In previous section we have estimated the presence of random-stochastic noises in Allan variance data, which affects the stability of atomic clocks. The Allan variance assumes the stationary of the clock error signal, a condition that is valid for ideal clocks only. For real clocks one has to pay attention in the evaluation of the clock stability, because even for short time intervals the clock can exhibit a non-stationary behavior. The possible reasons for the lack of stationary are temperature, humidity and other physical quantities that have a direct influence on clock stability behavior depending on construction of clock. Figure 4 shows the Allan variance plot of one of engineering model of rubidium atomic clock at ground testing, whose temperature profile variation is according to figure 5. It can be observed that at higher tau the stability profile is not as per inherent characteristic of rubidium clock behavior, indicating the non- stationary behavior of clock. Here assuming that the RW noise is less dominating than temp variations.

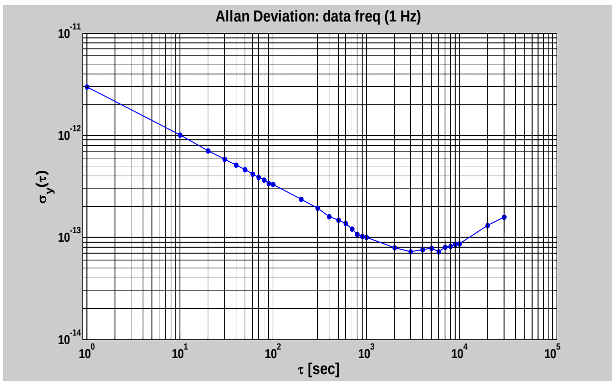


Figure 4: Allan deviation of a rubidium clock

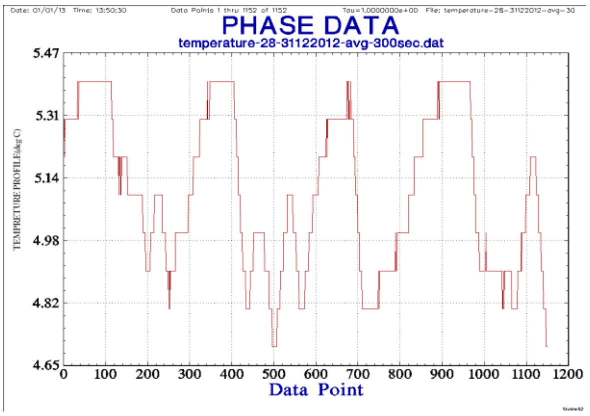


Figure 5: Temperature variation profile of rubidium clock

To analyze the effect of non-stationary noise due to temperature variations, we have adopted frequency domain method using power spectral density (PSD) of Allan variance data, which will follow power law model as discuss earlier.

A. Proposed algorithms for estimation of periodic noises

The proposed estimation algorithm begins with estimation of theoretical power spectral density using least square normalized-error (LSNE) regression method. Here the estimation of k and α is for complete measurement range, not decade wise. The reason behind estimation of k and α for complete measurement range is presence of spurious in complete frequency measurement range due to periodic noises in time domain data. Fig 6 shows the plot of estimated psd vs actual psd of one of the existing engineering model of rubidium atomic clock data at ground testing.

Let $y_{estimated}$ is the expected line of the estimated psd. Then

$$y_{estimated} = k * e^{-j\alpha} \tag{9}$$

$y_{estimated}$ is point where error is minimized using least square regression method

$$\grave{o}^2 = \min\left(\frac{(y_{actual} - y_{estimated})}{y_{actual}}\right) \tag{10}$$

Value of k is estimated by using unaveraged point of psd data.so

$$\text{k} = \frac{\text{sum}(\text{unaveraged point of psd data})}{\text{Total number of psd points}} \tag{11}$$

Then the value of α is estimated by

$$\alpha = \arg\left(\log \frac{y_{\text{estimated}}}{k}\right) \tag{12}$$

In order to estimate the spurious in the clock psd, we have first calculated the difference between estimated psd and actual psd of

$$€_{psd} = psd_{(actual)} - psd_{(estimated)} \tag{13}$$

To detect the spurious, we have to first select the threshold of estimated error .The selection of threshold is based upon the error profile of difference psd $€_{psd.}$ Fig 7 shows the spurious profile of Rubidium atomic clock phase data.

Figure 6: Estimated Power Spectral Density Vs Calculated Power Spectral Density

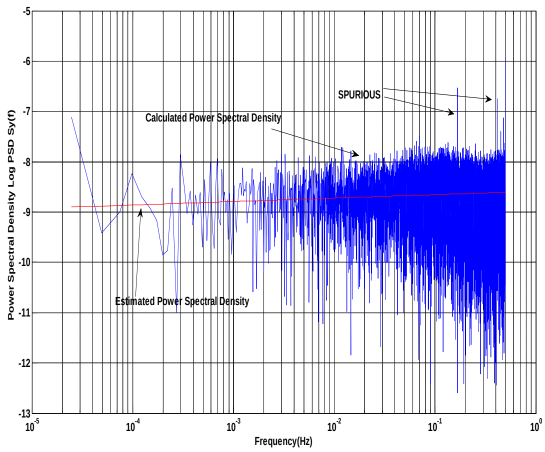
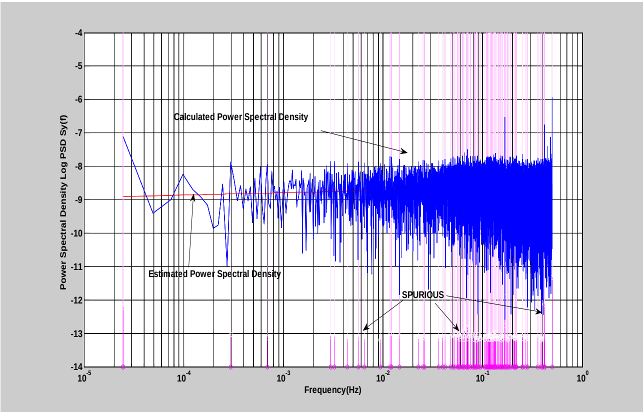


Figure 7: Spurious profile for rubidium clock data

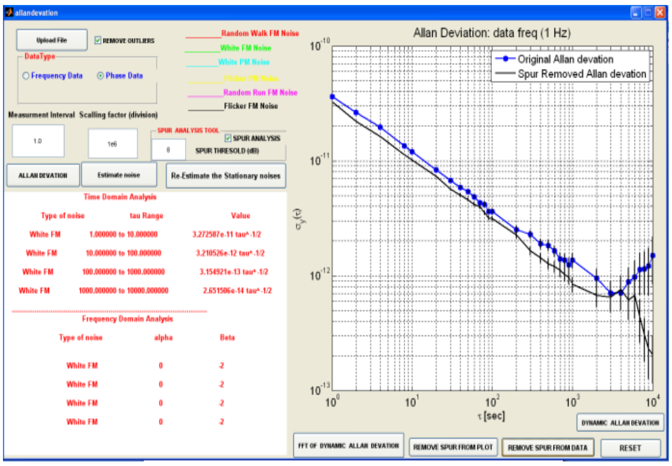


To analyze the effect of spurious on the stability of atomic clock, we have replaced the psd of actual data with estimated psd data at the
points, where we have detected the spurious. This corrected psd data is converted again in time domain frequency data in order to analyze the stability of atomic clock. This conversion is done using inverse frequency transform (IFFT) of power spectral density (psd) data. However normalization of psd data is required before applying IFFT. The spurious removed original data can be calculated as

$$Data_{(spurious\ removed)} = \left| \sqrt{IFFT\left(\frac{psd_{corrected}}{length\ of\ psd * sampling\ interval} \right)} \right| \qquad (14)$$

Figure 8 shows comparative Allan variance plot of original and spurious removed frequency data of rubidium atomic clock. It can be seen that after removing the spurious due to temperature variation, stability of rubidium atomic clock is improved and behavior is also as per expected specification. In order to re-validate our algorithm, we have taken another engineering model rubidium clock data set, whose spurious profile shows in figure 9, clearly reflects the presence of non-stationary noise. Figure 10 shows the improvement in stability of rubidium clock after removing the spurious in clock data. Another important aspect is that the noise profile of stochastic noise is change after removing the spurious from raw data. A provision is made in GUI to re-estimate stochastic noises.

Figure 8: Comparative Allan variance Plot



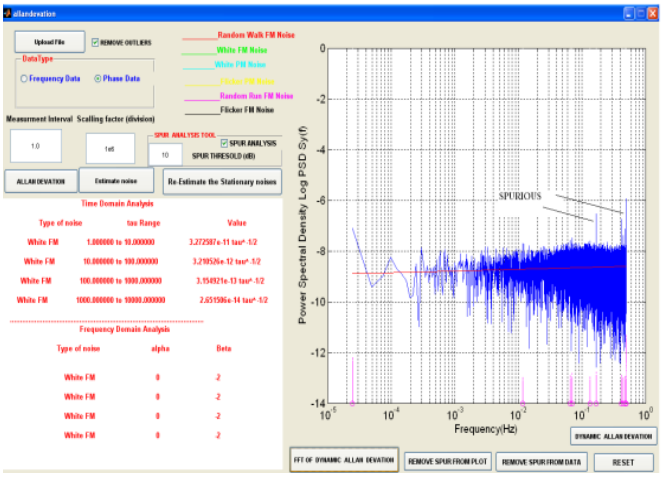


Figure 9: Spurious profile of Rubidium clock data

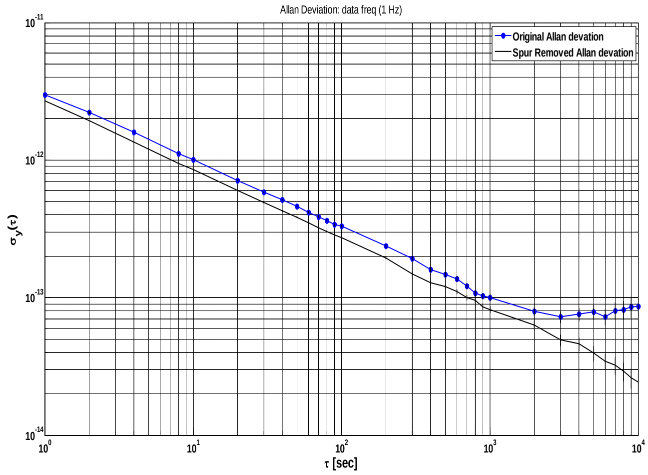


Figure 10: Comparative stability plot of Rubidium clock data[2]

IV. CONCLUSION

In this paper, we have predicted the stability behavior of rubidium atomic clock assuming the presence of random-stochastic noises and periodic noises. The simulation model is based on LSNE algorithm and the clock noise model is generated from various measured phase data of rubidium atomic clock. The result shows that stability of rubidium atomic clocks improves and also behaves as per specification after removing the periodic noises from various measured phase or frequency data of rubidium atomic clock. The fit technique is less complex and does not have convergence issues compared to iterative logarithmic methods that are typically used for fitting Allan variance data.

A user friendly GUI is developed, which analyzes both random-stochastic noises and periodic noises from rubidium atomic clock data.

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